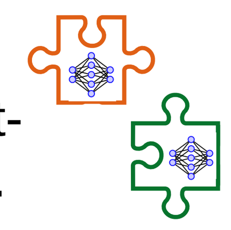
Compositional Learning of Dynamical System Models Using Port-Hamiltonian Neural Networks

Cyrus Neary and Ufuk Topcu
The University of Texas at Austin

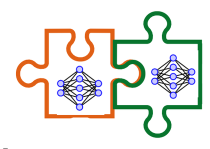
TEXAS The University of Texas at Austin — CENTER FOR autonomy

The central question.
How can we leverage physics-based knowledge to build **compositional** neural network models of dynamical systems?

Independently learn component-level submodels.



Compose the submodels to make system-level predictions.



A summary of the approach.
Enforce *port-Hamiltonian* structure on neural ODEs representing the subsystems and the composite system.

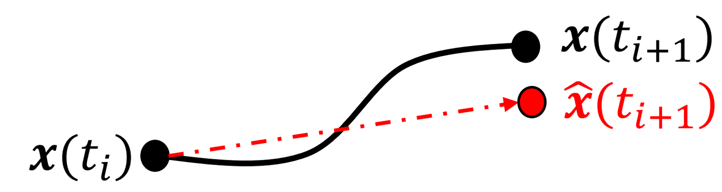
1. Parametrize and train the subsystem models independently.
2. Develop a framework to **compose** the learned submodels.
3. Leverage port-Hamiltonian structure to provide **guarantees** of useful model properties.

Port-Hamiltonian neural networks

Given: Dataset of trajectories $\{(x(t_1), u(t_1)), \ldots, (x(t_n), u(t_n))\}$.

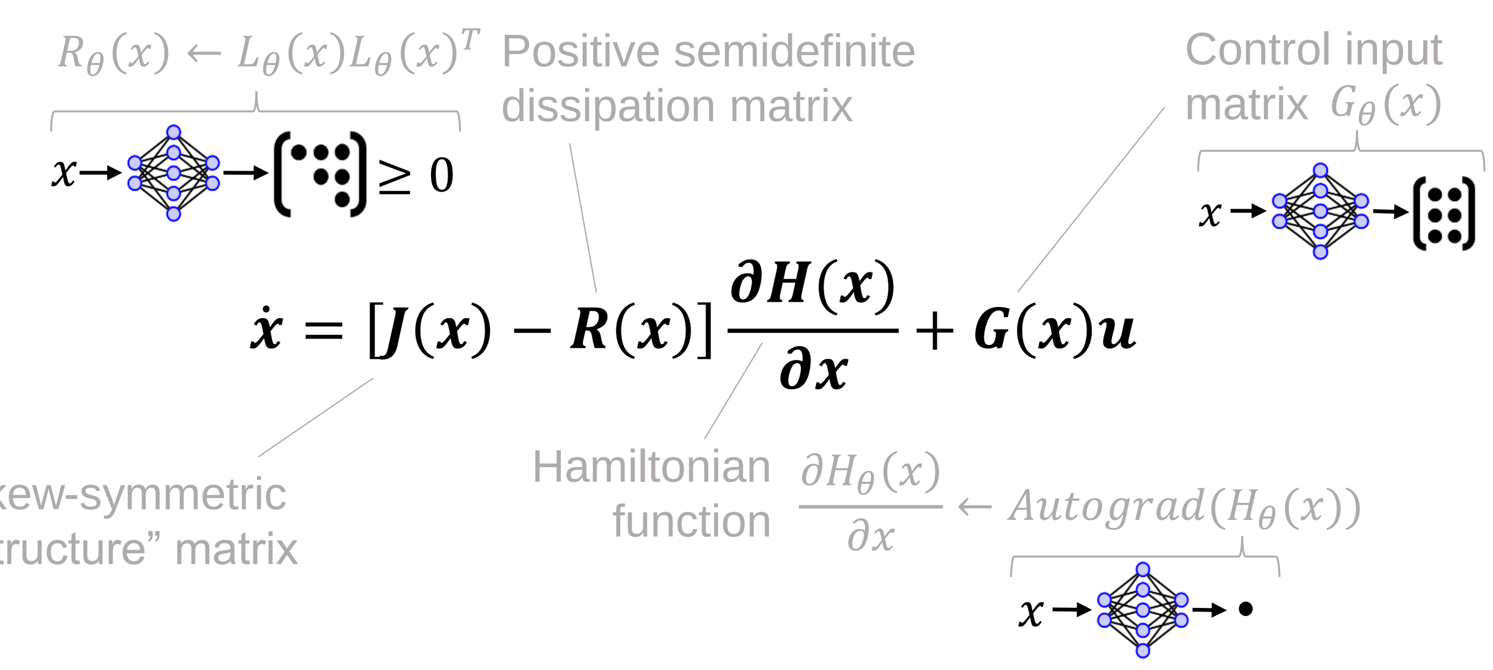
System state — Control input

Objective: Learn to predict $x(t_{i+1})$ from $x(t_i), u(x(t_i), t_i)$.



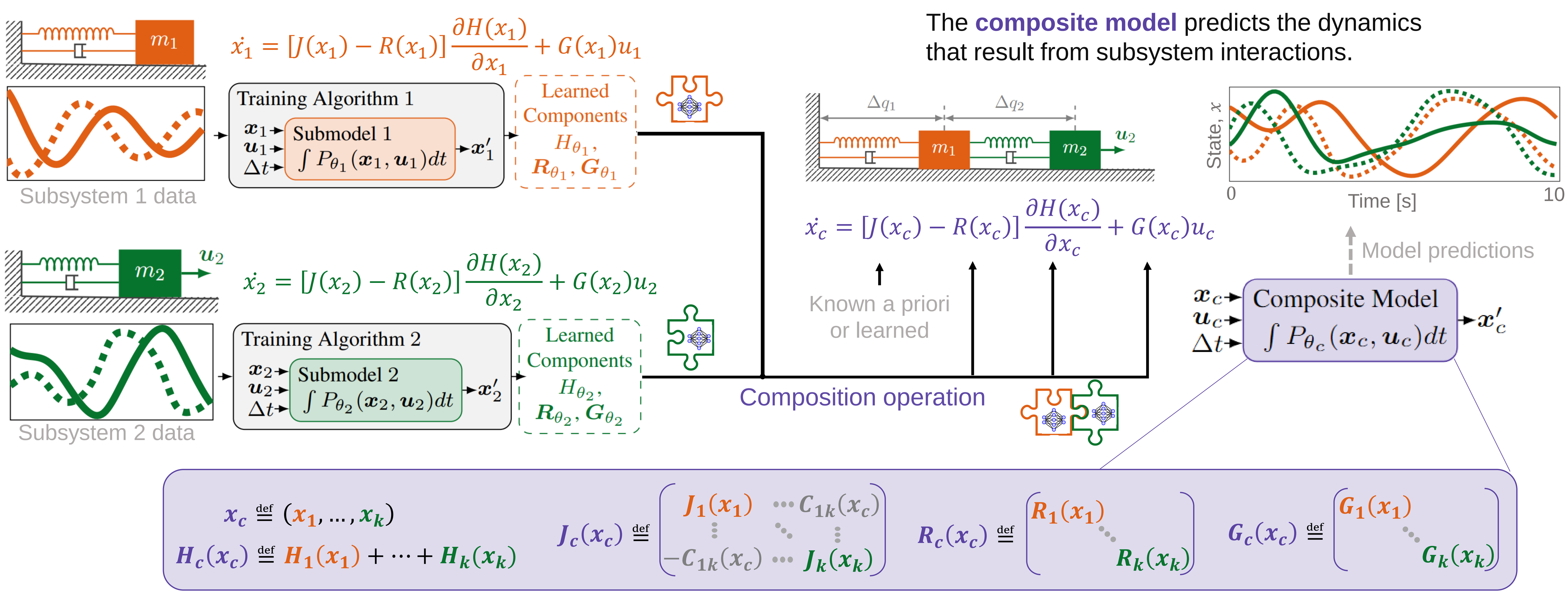
The Model: Numerically integrate a parametrized ODE.

$$\dot{x} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x} + G(x)u$$



Composing port-Hamiltonian neural networks

The **composite model** predicts the dynamics that result from subsystem interactions.



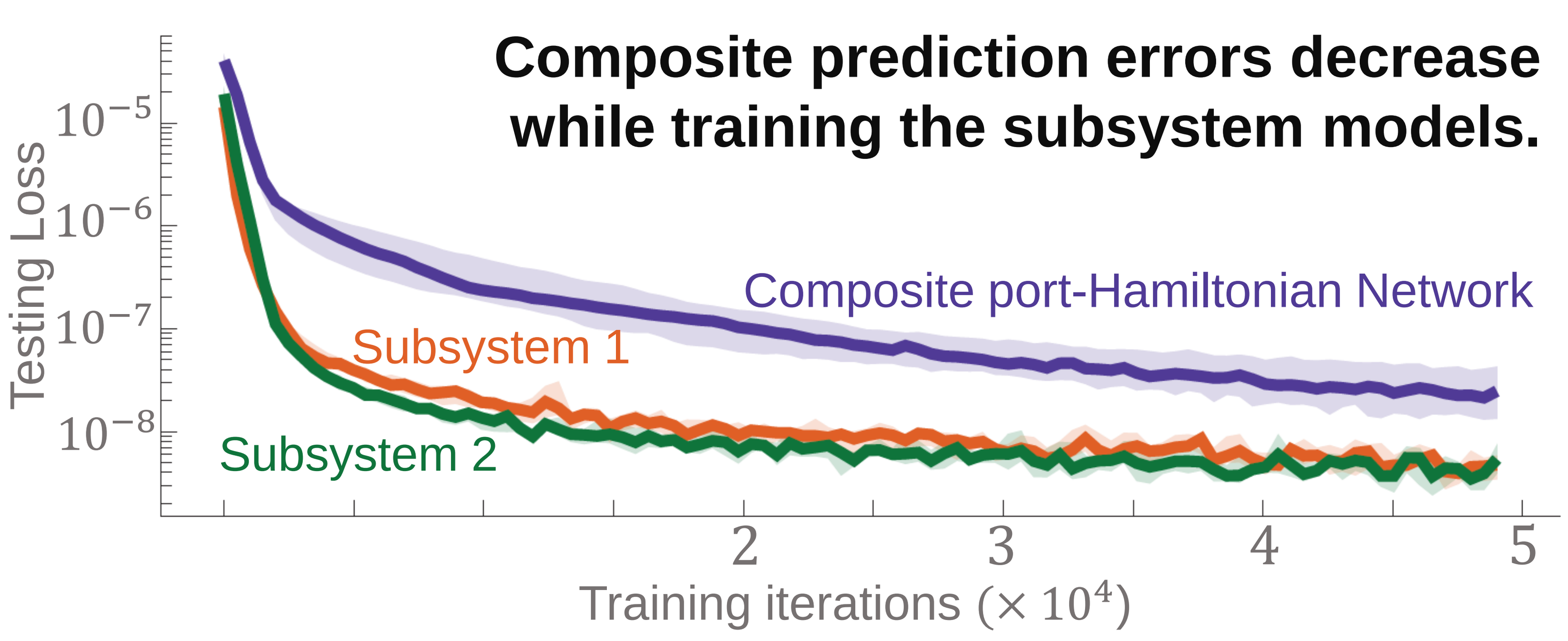
Off-diagonal composition terms $C_{ij}(x_c)$ can be derived from engineering knowledge or learned from data generated by the composite system.

Theorem: Prediction error of **composite port-Hamiltonian network** is bounded by errors of subsystem models and errors of composition terms.

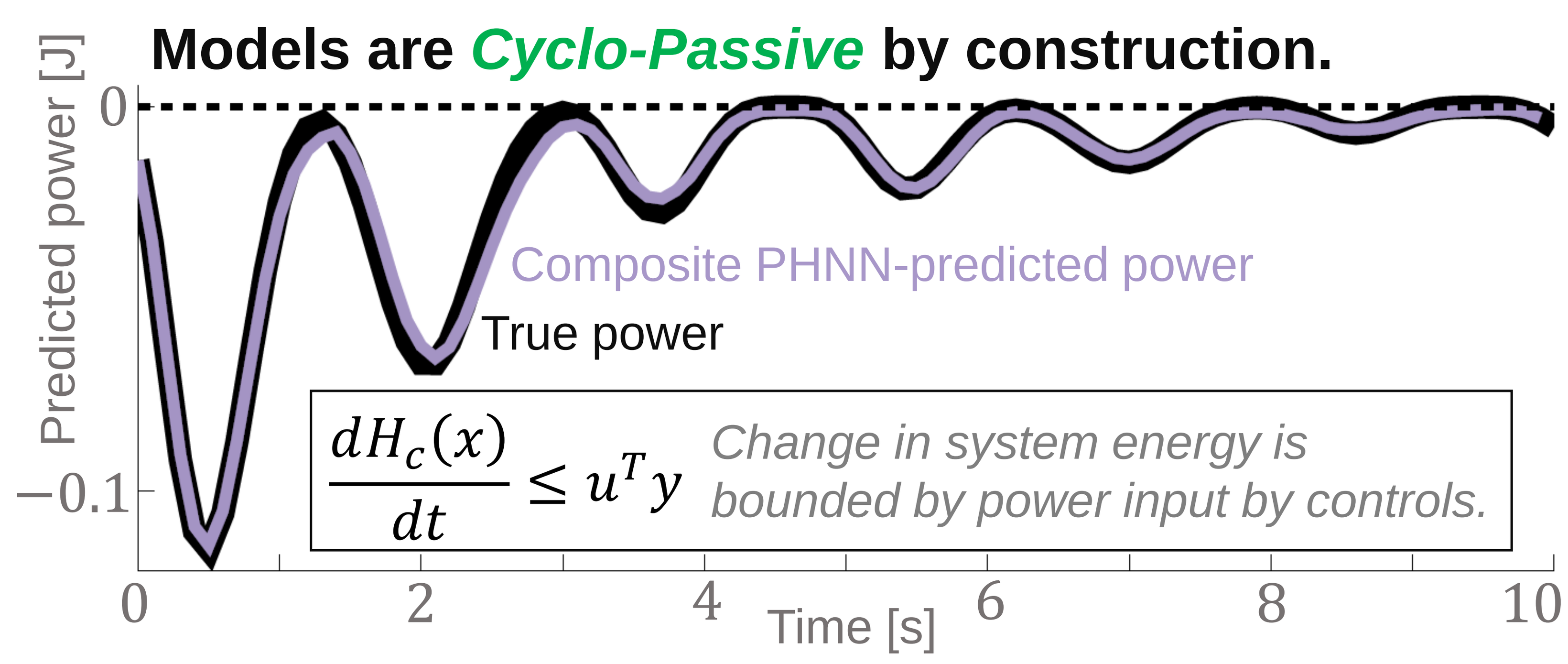
$$Err_{comp} \leq \sum_{i=1}^{k}\left[\varepsilon_i + 2\sum_{j>i}^{k}\gamma_{ij} + \sigma_{ij}\eta_j\right]$$

Composite model error — Subsystem errors — Errors introduced by coupling terms — Errors introduced by Hamiltonian terms

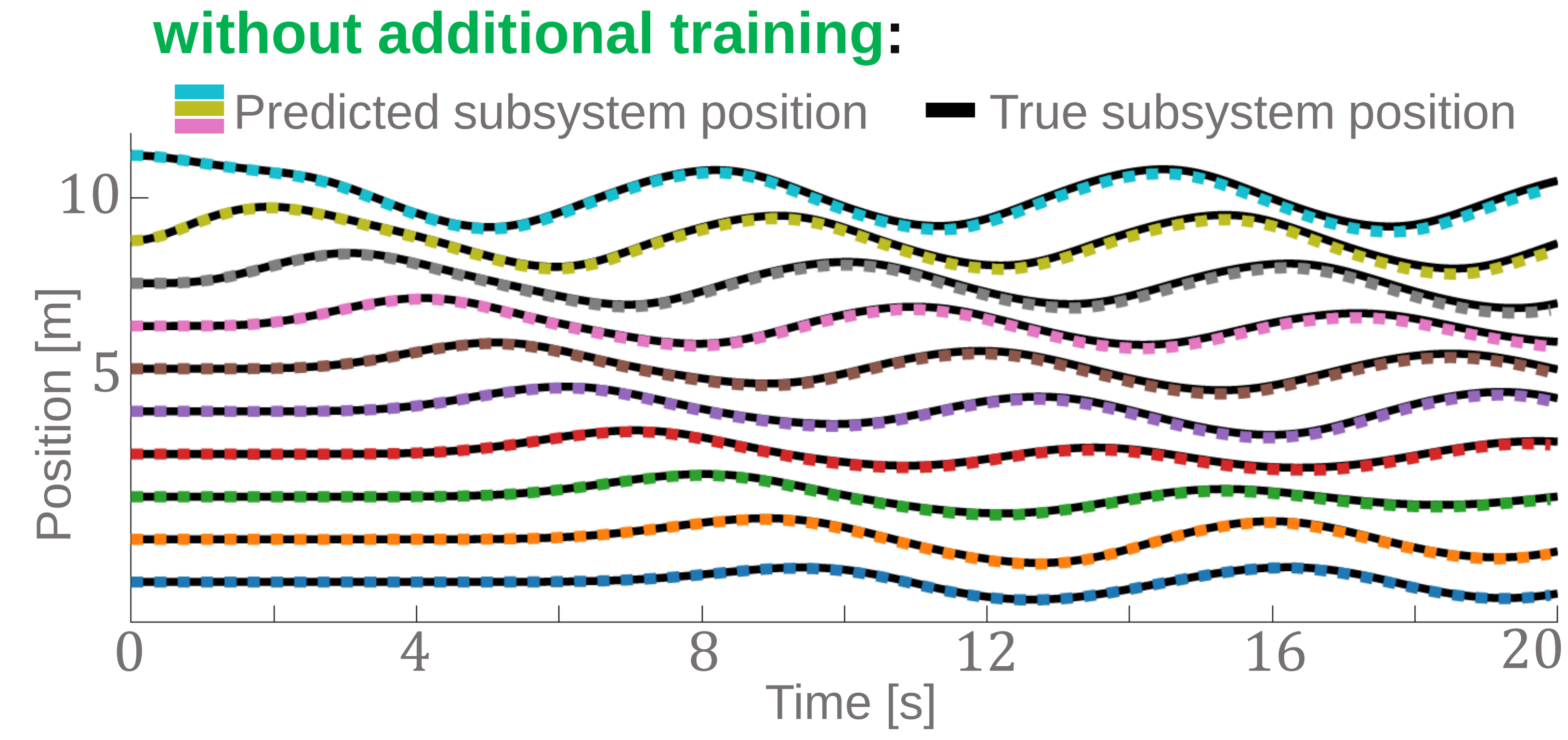
Composite prediction errors decrease while training the subsystem models.



Models are *Cyclo-Passive* by construction.



Modular interconnection of ten subsystems without additional training:



cneary@utexas.edu